B.C.A. Semester-III (Honours) Examination, 2022-23

BACHELOR OF COMPUTER APPLICATION

Course ID : 33314 **Course Code : BCA/GE-03**

Course Title : Mathematics-II

Time : 3 Hours Full Marks : 80

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Choose the correct option for the following questions: 1×10=10

a) The value of $\lim_{x\to 0}\frac{1}{2+e^{\frac{1}{x}}}$

i) is 0 ii) is 1

iii) is –1 iv) does not exist

b) If $f(x)$ be continuous function and $g(x)$ be a discontinuous function then $f(x)+g(x)$ is

i) Continuous

ii) Discontinuous

iii) Continuous at some points

iv) Discontinuous at some points

c) The degree of the homogeneous function

$f(x, y) = \tan^{-1}\frac{y}{x}+\sin^{-1}\frac{x}{y}$ is

i) 0 ii) –1

iii) 1 iv) 2

d) Every convergent sequence is

i) monotonic

ii) bounded

iii) unbounded

iv) with non-unique limit

e) The value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ is

i) 0 ii) –1

iii) $\frac{\pi}{4}$ iv) $\frac{\pi}{2}$

f) The improper integral $\int_0^{\infty}\sin mx\,dx$

i) Converges ii) Diverges

iii) Does not exist iv) is equal to 2

g) If $\lim_{n\to\infty} x_n = l$, then $\lim_{n\to\infty} \frac{x_1 + x_2 + \ldots + x_n}{n} =$

i) ∞ ii) $-\infty$

iii) 0 iv) l

h) The infinite series $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}$ is

i) Divergent

ii) Convergent

iii) Conditionally convergent

iv) Absolutely divergent

i) The complimentary function (C.F.) of the ODE

$\frac{d^2y}{dx^2}+8\frac{dy}{dx}+25y=0$ is

i) $y = e^{-4x}(A \cos 3x + B \sin 3x)$

ii) $y = e^{-3x}(A \cos 4x + B \sin 4x)$

iii) $y = e^{4x}(A \cos 3x + B \sin 3x)$

iv) $y = Ae^{-4x}(\cos 3x \sin 3x)$

j) The degree of the ODE

$\cos\left(\frac{d^2y}{dx^2}\right)+\tan^{-1}\left(\frac{dy}{dx}\right)+4xy=0$ is

i) 0 ii) 2

iii) 1 iv) Undefined

GROUP–B

2. Answer any **ten** of the following questions: 2×10=20

a) Define Cauchy sequence.

b) Show that the sequence $\{n+(-1)^n. n\}_n$ oscillates infinitely.

c) State Reimann's Rearrangement Theorem.

d) State the order and degree of the ODE:

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{2}{3}}=\frac{d^2y}{dx^2}.$$

e) Calculate $(D^2 - 3D + 4) \sin x$.

f) Define first order linear ordinary differential equation (ODE).

g) Evaluate: $\int \frac{\sqrt{x}dx}{x(x+1)}$.

h) Evaluate: $\int_{-2}^{2}|1-x^2|dx$.

i) When an integral is called improper?

j) State Maclaurin's theorem with Cauchy's form of remainder.

k) Define homogeneous function with an example.

l) Examine the equality of f_{xy} and f_{yx} where $f(x, y) = x^3y + e^{xy^2}$.

m) If $\lim_{x\to c} f(x) = l$, then show that $\lim_{x\to c} |f(x)| = |l|$.

n) State Intermediate Value property of continuous functions.

o) If $y = (\sin x)^3$, find the n th derivative y_n.

GROUP–C

3. Answer any **four** of the following questions: 5×4=20

a) Let $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ when $x \neq 0$ and $f(0) = 0$. Show that f is derivable at $x = 0$ but $\lim_{x\to 0} f'(x) \neq f'(0)$.

b) State Rolle's Theorem and give its geometrical significance.

c) Integrate: $\int \log\left(x+\sqrt{x^2+a^2}\right)dx$.

d) Solve: $(D^2 + 1)y = \cos x$.

e) Show that the sequence $\{S_n\}_n$, defined by the recursion formula $S_{n+1} = \sqrt{3S_n}, S_1 = 1$, converges to 3.

f) If b > 0, then show that the series

$$x + \frac{a-b}{2!}x^2 + \frac{(a-b)(a-2b)}{3!}x^3 + \ldots \text{ converges}$$

absolutely for $|x| < b^{-1}$.

GROUP–D

4. Answer any **three** of the following questions: 10×3=30

a) i) If $y = 2x - \tan^{-1}x - \log\left(x+\sqrt{1+x^2}\right)$, show that y continuously increases as x changes from zero to positive infinity.

ii) If $y = \frac{x^2+x-1}{x^3+x^2-6x}$, find the n th derivative of y.

iii) If $V=ax^2+2hxy+by^2$, then show that $V_x^2V_{yy} - 2V_xV_yV_{xy} + V_y^2V_{xx} = 8(ab-h^2)V$. 3+3+4

b) i) Show that: $\frac{v-u}{1+v^2} < \tan^{-1}v - \tan^{-1}u < \frac{v-u}{1+u^2}$, if 0<u<v.

ii) Solve:

$(x^2D^2+xD-1)y = \sin(\log x) + x\cos(\log x)$,

where $D \equiv \frac{d}{dx}$.

c) i) Test the convergency of the integral $\int_0^{\pi} \frac{dx}{1-\cos 2x}$.

ii) Evaluate: $\int \sqrt{\frac{a+x}{a-x}}\, dx$.

iii) Find the limit of the sum:

$\lim_{n\to\infty} \left\{ \frac{n!}{(kn)^n} \right\}^{\frac{1}{n}}, \quad k \neq 0$ 3+3+4

d) i) Use Cauchy's General Principle of Convergence to show that the following sequence

$$\left\{1-\frac{1}{2}+\frac{1}{3}-\ldots+\frac{(-1)^{n-1}}{n}\right\}_n.$$

ii) State limit form of the Comparison Test. Test the convergency of the series

$$\sum \frac{n^2-1}{n^2+1} x^n, x > 0.$$

e) i) Show that the function $f(x) = [x] + [-x]$, where $[x]$ denotes the greatest integer not exceeding x, has removable discontinuity for integral values of x.

ii) If $f(x)$ and $g(x)$ are differentiable in the interval (a, b) then prove that there is a number x, $a < \xi < b$, such that

$$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a) \begin{vmatrix} f(a) & f'(\xi) \\ g(a) & g'(\xi) \end{vmatrix}.$$

f) i) Solve the ODE: $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$.

ii) Prove that every monotonically increasing sequence which is bounded above is convergent.

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